TRIBHUVAN UNIVERSITY
Institute of Science and Technology
Bachelor of Science in Computer Science and Information Technology
Model Question Paper

Bachelor Level/ First Year/ First Semester/ Science — Full Marks: 80
Computer Science and Information Technology (MTH 104) — Pass Marks: 32
(Calculus and Analytical Geometry) — Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.

Attempt all questions.

Group A (10x2=20)

1. Verify Rolle's theorem for the function $y = \sqrt{1 - x^2}$ on [-1, 1] and hence find the corresponding point.
2. Find the length of the curve $y = \frac{1}{3}(x^2 + 2)^{\frac{3}{2}}$ from $x = 2$ to $x = 3$.
3. Test the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ for p a real constant.
4. Find the polar equation of the circle $x^2 + (y - 3)^2 = 9$.
5. Find a spherical coordinate equation for $x^2 + y^2 + z^2 = 4$.
6. Use double integral to find the area of the region bounded by $y = x$ and $y = x^2$ in the forst quadrant.
7. Verify the Euler's theorem for mixed partial derivatives: $w = x \sin y + y \sin x + xy$.
8. Use the chain rule to find the derivative of $w = xy$ with respect to t along the path $x = \cos t, y = \sin t$.
9. Form a partial differential equation by eliminating the constants a and b from the surface $(x - a)^2 + (y - b)^2 + z^2 = c^2$.
10. Solve the partial differential equation $p + q = x$, where the symbols have their usual meanings.

Group B (5x4=20)

11. State and prove the mean value theorem for definite integral. Apply the theorem to calculate the average value of $f(x) = 4 - x^2$ on [0, 3].

12. Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos\theta$.

13. Find the curvature and principal unit normal for the helix $r(t) = (a\cos t)i + (a\sin t)j + (bt)k$ with $a, b \geq 0$, and $a^2 + b^2 \neq 0$, where the symbols have their usual meanings.

14. What do you mean by directional derivative in the plane? Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point (2, 0) in the direction of te vector $\vec{A} = 3\vec{\imath} - 4\vec{\jmath}$.

15. Find a particular integral of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial z}{\partial y} = 2y - x^2$.

Group C (5x8=40)

16. Graph the function $y = x^{5/3} - 5x^{2/3}$.

17. Find the Taylor's series and the Taylor's polynomial generated by $f(x) = e^{ax}$ and $g(x) = x\cos x$ at $x = 0$.

18. Evaluate the double integral $\int_0^4 \int_{x=\frac{y}{2}}^{x=\frac{y}{2}+1} \frac{2x-y}{2}\, dx\, dy$ by applying te transformation $u = \frac{2x-y}{2}$, $v = \frac{y}{2}$ and integrating over an appropriate region in the uv-plane.

    OR

    Find the volume of the region D enclosed by $z = x^2 + 3y^2$ and $= 8 - x^2 - y^2$.

19. Find the local minima, local maxima and saddle points of the function $f(x, y) = 2xy - 5x^2 - 2y^2 + 4x + 4y - 4$.

    OR

    Find the maximum and minimum of the function $f(x, y) = 3x - y + 6$ subject to the constraint $x^2 + y^2 = 4$ and explain its geometry.

20. Show that the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$, $c^2 = \frac{T}{\rho}$ is

$$u(x,t) = \frac{1}{2}[f(x+ct) + f(x-ct)] + \frac{1}{2c}\int_{c-ct}^{x+ct} g(s)ds$$

And deduce the result if the initial velocity is zero.